

Benefit of advanced lunar tracking for determining the parameters of the Earth-Moon system

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Abstract

The Earth-Moon distance has been measured with Lunar Laser Ranging (LLR) since 1970. In recent years, there have been improvements in both, lunar tracking and data analysis. Supported by tracking in infrared, the measurements have achieved a higher accuracy, the number of NPs per night is higher compared to the years before 2015 and the NPs are better distributed over the lunar orbit and retro-reflectors. Together with improvements in the LLR modelling and analysis software, the determination of various parameters in the Earth-Moon system is now possible with higher accuracy. Furthermore, an advanced lunar-tracking technique, called Differential Lunar Laser Ranging (DLLR), will be realized at the Table Mountain Observatory (TMO) of JPL in the near future. There, a novel kind of observable, i.e., differenced lunar ranges, can be obtained reaching an extreme high precision of $\sim 30 \mu\text{m}$.

By analysing LLR data, Earth Orientation Parameters (EOPs) such as the Earth rotation phase ΔUT1 and the terrestrial pole coordinates can be determined along with other parameters of the Earth-Moon system in a least-squares adjustment. Focusing on ΔUT1 and terrestrial pole coordinates from different LLR constellations, such as single or multi-station data and for different numbers of NPs per night, the accuracies of the estimated Earth rotation phase and pole coordinates from the new LLR data have improved significantly compared to previous results and achieve an accuracy of $20 \mu\text{s}$ for ΔUT1 , 0.52 mas for x_p and 0.66 mas for y_p from subsets of the LLR time series with 15 normal points per night.

In addition, the potential of DLLR is investigated using simulated data, with the focus on combining LLR and DLLR to benefit from the advantages of both techniques. After combination, the estimation of the lunar orientation-, rotation- and interior-related parameters are significantly improved even when only using DLLR data over a rather short time span.

1. Introduction

Lunar Laser Ranging (LLR) as the measurement of the Earth-Moon distance with laser pulses is now possible for more than 53 years. LLR data are obtained from six observatories: the Côte d'Azur Observatory (OCA) in France, the Apache Point Observatory Lunar Laser ranging Operation (APOLLO) in USA, Lunar Ranging Experiment Observatory (LURE) in USA, the McDonald Laser Ranging Station (MLRS) in USA, the Matera Laser Ranging Observatory (MLRO) in Italy and the Wettzell Laser Ranging System (WLRS) in Germany. On the Moon there are five retro-reflectors, i.e., A11, A14, A15, L1 and L2 (deployed during APOLLO 11, 14, 15 missions and Luna 17, 21 missions) that can be tracked by the observatories. Via the analysis of the LLR data, contributions are possible to reference frames (for Earth, Moon and inertial), to the determination of Earth Orientation Parameters (EOPs),

relativity tests (Biskupek et al., 2021; Zhang et al., 2020), the better understanding of the lunar interior and so on.

The recent dataset includes 30172 normal points (NPs) over the time span April 1970 to April 2022. As of 2015, many NPs were measured with laser pulses at IR wavelength, enabling distance measurements near new and full Moon (Chabé et al., 2020) for OCA and WLRs. This leads to a better coverage of the lunar orbit over the synodic month, i.e., the time span in which Sun, Earth, and Moon return to a similar constellation again. With a better coverage of the lunar orbit, it is possible to perform a more uniform estimation of various parameters of the Earth-Moon system with higher accuracy. This benefit together with a better distribution of the measurements over the five retro-reflectors and a higher number of NPs per night give the motivation for the new determination of EOPs from LLR.

For lunar tracking, in addition to the improvements on the LLR data, a new technique, called Differential Lunar Laser Ranging (DLLR), will be implemented at the Table Mountain Observatory (TMO) of JPL. The differenced lunar ranges as a new kind of observable can be obtained by differencing the successive range measurements which are taken from a station with its fast switching between different reflectors on the Moon. By largely reducing the atmospheric error, which is a major error source in LLR, a value of about 30 μm can be achieved for the accuracy level of DLLR (Dehant et al., 2017). DLLR is expected to significantly improve the knowledge of the lunar orientation, rotation and interior, which is a DLLR focus (Turyshev et al., 2018). To show the benefit of DLLR after a few years of dedicated measurements, simulated DLLR data with a short time span are combined with LLR data over a long time span. The latter are needed to get a good lunar orbit as basis.

2. Earth Rotation Parameter Estimation

The terrestrial pole coordinates, x_p and y_p , describe the change of the rotation axis in relation to the Earth's surface. The Earth rotation phase ΔUT1 and the Length of Day (LOD) refer to the rotation of the Earth about its axis. All these parameters are summarised as Earth Rotation Parameters (ERPs). Together with the celestial pole

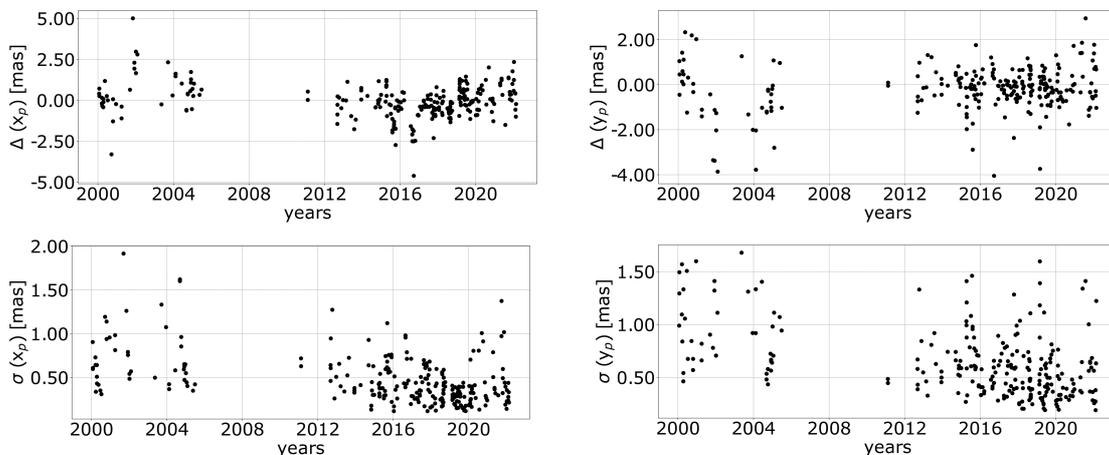


Figure 1: Differences to the a-priori IERS C04 EOP series for the pole coordinates x_p and y_p (top figures) and their respective uncertainties given as the formal error from the least-squares adjustment (bottom figures).

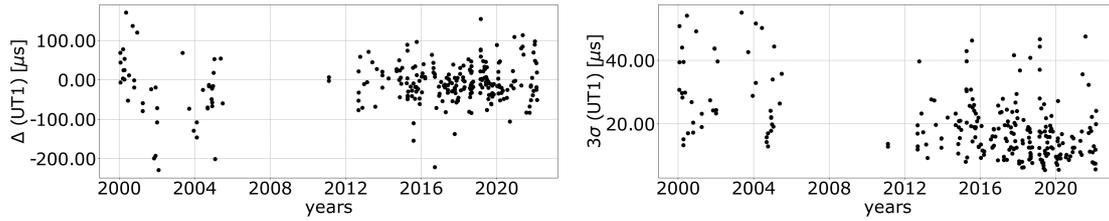


Figure 2: Differences to the a-priori IERS C04 EOP series for the Earth rotation phase $\Delta UT1$ (left figure) and the uncertainties given as 3 times the formal error from the least-squares adjustment (right figure).

offsets as corrections to the conventional precession–nutration model, they define the EOPs. As the rotation matrix between the Earth fixed International Terrestrial Reference System (ITRS) and the space fixed Geocentric Celestial Reference System (GCRS) includes EOPs in its calculation, these can be estimated from LLR analysis, as shown, e.g., by Biskupek et al. (2022), Singh et al. (2022), Hofmann et al. (2018), Biskupek (2015).

The whole data set of NPs is pre-analysed for ERP determination in the LLR analysis, where different configurations are considered. It is possible to estimate ERPs from the data of all observatories or only for a single observatory. It is also possible to vary the number of NPs per night or to choose specific wavelengths. For the current study, only NPs from the OCA observatory starting in January 2000 were considered. The minimum number of NPs per night was set to 15. These conditions give 257 nights for which ERPs could be determined. The strategy for the ERPs determination is to use all LLR NPs to determine the parameters of Earth-Moon system and only the NPs for a specific night, to calculate the ERPs for that night. The ERPs for nights, that are not considered in the fit, because the above conditions were not met, are fixed to the a-priori IERS C04 EOP series. The velocities of the observatories are fixed to ITRF2014 values. The results for the ERPs are given in Figure 1 and Figure 2 with the differences to the IERS C04 EOP series for the respective components and their uncertainties, that are given as the formal error from the least-squares adjustment. From the uncertainties of the individual nights the weighted root mean square (wrms) is calculated. The number of NPs per night is used as the weight. For the x_p component, the wrms, as resulting accuracy, is 0.52 mas and 0.66 mas for the y_p component. Using the Earth radius at the equator as 6378 km, these results correspond to 1.56 cm for x_p and 1.98 cm for y_p as spatial resolution on the Earth’s surface. For $\Delta UT1$ the wrms of the uncertainties is calculated in the same way as for the pole coordinates, but here the formal errors from the least-squares adjustment are multiplied with a scaling factor of three. The scaling factor was investigated based on sensitivity studies and error analysis. The wrms and the resulting accuracy for $\Delta UT1$ is 20.1 μs . That corresponds to 9.2 mm on the Earth’s surface. Compared to previous ERP results from LLR (Hofmann et al. 2018) the uncertainty improves by a factor of two for the Earth rotation phase, because of the benefits from the IR measurements by OCA. Further investigations are performed, to analyse whether the scaling factor for the uncertainties of $\Delta UT1$ is still valid and needed in the LLR analysis.

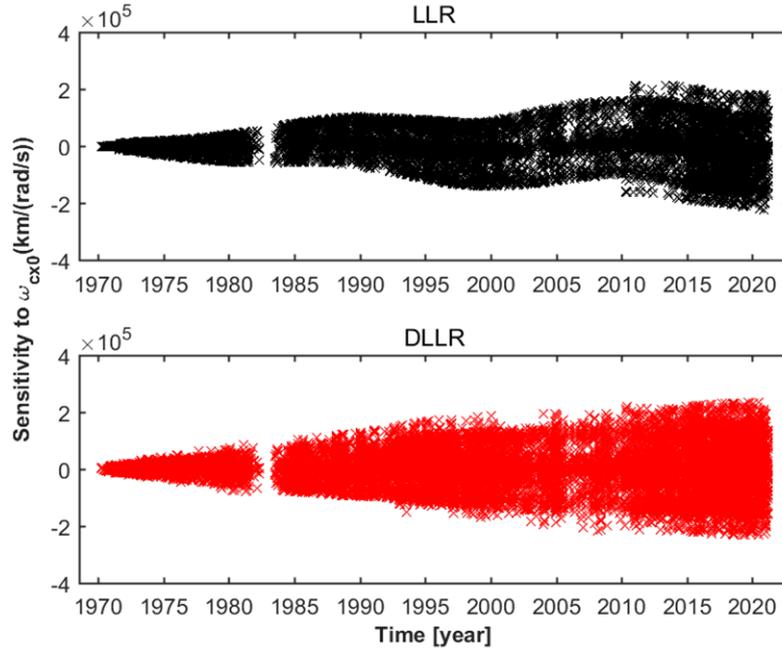


Figure 3: LLR (upper panel) and DLLR (lower panel) sensitivity to ω_{cx0} .

3. Differential Lunar Laser Ranging

In a previous research paper, Zhang et al. (2022) investigated DLLR characteristics using simulated DLLR data with the same time period (1970-2021, more than 50 years) and the same amount of data as the real LLR data used. They found that DLLR has similar sensitivity to LLR for some parameters, e.g., the orientation, rotation and interior parameters of the Moon (see Figure 3 for the initial x-component of the lunar core angular velocity at Julian date 2440400.5 ω_{cx0} as an example). However, DLLR is much less sensitive for some other parameters, e.g. the orbital elements (position and velocity) of the Moon, but this is balanced by the expected extremely high DLLR measurement precision of 30 μm . Moreover, compared to LLR, DLLR has a very big advantage in estimating parameters such as lunar orientation, rotation and interior parameters. However, for some other parameters, e.g., the lunar orbital elements, DLLR cannot contribute as much.

DLLR data with a time span of more than 50 years are not available in the near future. In order to still be able to use the advantages of DLLR shown in Zhang et al. (2022), one can combine DLLR and LLR. In this way, the disadvantages of the short DLLR time series can be compensated by the very long time span of LLR data, where its high-precision lunar orbit is of biggest relevance. The basic information on the data used to

Table 1: Basic information of the used data

Data type	Time span	Data amount	Observatory	Reflector/Reflector baseline
real LLR	1970-2021	28093	MLRS, LURE, WLRS, OCA, APOLLO, MLRO	A11, A14, A15, L1, L2
simulated LLR	2022-2026	8665	MLRS, LURE, WLRS, OCA, APOLLO, MLRO	A11, A14, A15, L1, L2
simulated DLLR	2022-2026	8665	MLRS, LURE, WLRS, OCA, APOLLO, MLRO	random baselines of A11, A14, A15, L1, and L2

Table 2: Parameter estimation from the combination of LLR and DLLR

Data type	Parameter accuracy					
	X_{m0} [km]	Y_{m0} [km]	Z_{m0} [km]	ω_{cx0} [rad/s]	ω_{cy0} [rad/s]	ω_{cz0} [rad/s]
LLR (real+simulated)	4.9×10^{-5}	2.0×10^{-5}	1.3×10^{-5}	2.0×10^{-11}	1.6×10^{-11}	1.2×10^{-10}
Combined LLR and DLLR	4.3×10^{-5}	1.6×10^{-5}	1.0×10^{-5}	1.6×10^{-12}	2.0×10^{-12}	2.1×10^{-11}

study the combination is presented in Table 1. Based on these data, parameters for the initial position of the moon (X_{m0}, Y_{m0}, Z_{m0}) and the initial angular velocity of the lunar core ($\omega_{cx0}, \omega_{cy0}, \omega_{cz0}$), among others, were determined. Obviously, the accuracies of the lunar orbital elements improve only slightly. In contrast, the accuracies of the initial angular velocity of the lunar core improve by about one order of magnitude, although only the DLLR data for a time span of five years were used (see Table 2).

4. Conclusions

ERPs can be determined from LLR data analysis. Here 15 NPs per night from OCA were used to determine the Earth rotation phase $\Delta UT1$ and the pole coordinates x_p and y_p as differences to the a-priori IERS C04 series. For $\Delta UT1$, the accuracy is 20.1 μs , for the x_p component it is 0.52 mas and 0.66 mas for the y_p component. The high-accurate IR data from OCA are very beneficial for the ERPs determination, because of their optimised distribution over the reflectors and synodic month as well as the higher number of NPs for one night. With more IR data from the observatories OCA and WLRS, it is expected that the parameters of the least-squares adjustment will further improve.

In addition, the new DLLR data will help to improve LLR results in the future by combining the two data types. The DLLR data will mainly contribute to a better determination of the parameters related to the lunar interior, rotation and orientation. In a further study, we will investigate whether the LLR and DLLR combination is also beneficial for the determination of ERPs.

Acknowledgements

We acknowledge with gratitude, that more than 53 years of processed LLR data have been obtained under the efforts of the personnel at McDonald Laser Ranging Station, USA, Côte d'Azur Observatory, France, Lure Observatory on Maui/Hawaii, USA, Apache Point Observatory Lunar Laser ranging Operation, USA, Matera Laser Ranging Observatory, Italy and Geodetic Observatory Wettzell, Germany. We also acknowledge with thanks the funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy (EXC-2123 QuantumFrontiers - Project-ID 390837967).

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